# CSC 150 LAB 10-1 NUMERICAL INTEGRATION

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We are going to write a function called `integrate`, that takes another function `fun`, and two real numbers `a` and `b` as parameters, and returns the definite integral of `fun` over the interval $[a, b]$.

That is, when we call `integrate(f, a, b)` for a function $f$, it returns the value $\int_a^b f(x)dx$.

## 1. Functions As Parameters

The ability to use functions as parameters is important in programs that solve equations to find the roots of functions, or have to find the definite integral of a function over some interval, or have to compute the derivative `f'(a)` of some function `f` at a point `a`.

As a simple example, consider a function

```
double evaluate(double f(double), double x)
{
   return f(x);
}
```

This function has two parameters,

(1) a function `f` that takes a parameter of a type double and returns a double, `double f(double)`
(2) a number `x` of type double at which to evaluate the function.

A call `evaluate(f, x)` simply returns the result of applying `f` to `x`.

Here is an example program which applies the `evaluate` function to evaluate two different functions: an identity function, and a squaring function.

```
#include <iostream>
using namespace std;

double evaluate(double fun(double), double x)
{
    return fun(x);
}

double identityFunction(double x)
{
    return x;
}

double squaringFunction(double x)
```

```
{
   return x*x;
}

int main()
{
    double number;
    cout << "Enter a number: ";
    cin >> number;

    cout << "The identity function returns "
         << evaluate(identityFunction, number) << endl;
    cout << "The squaring function returns "
         << evaluate(squaringFunction, number) << endl;
}
```

The result of running the program looks like this

```
Enter a number: 1.2
The identity function returns 1.2
The squaring function returns 1.44
```

## 2. Exercise 1

Modify the above program by adding a function `printValues`

```
void printValues(double f(double), double a, double b, int n)
```

that takes as parameter another function $f$, two real numbers $a$ and $b$ defining a closed interval $[a, b]$, and an integer $n$.

The function prints the values of $f$ in the interval $[a, b]$ in interval increments of size $\frac{b-a}{n}$.

Modify your main function so it asks the user for the numbers $a, b$ that define the closed interval, and an integer $n$, and prints the values for both the identify and squaring functions. Here is a sample run

```
Enter the lower and upper bounds of an interval: 0 5
Enter the number of subintervals to use: 10
Identity function values in the interval are
0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5
Square function values in the interval are
0, 0.25, 1, 2.25, 4, 6.25, 9, 12.25, 16, 20.25, 25
```

Your program needs to match this sample output, down to the use of commas to separate the printed values.

## 3. Numerical Integration

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We can estimate the definite integral $\int_a^b f(x)dx$ of a function by estimating the area under the graph of the function. We do this by by dividing the interval $[a, b]$ into $n$ subintervals, and estimating the area under a function over a subinterval $[x, x + \Delta]$

by a rectangle of height $f(x)$ and width $\Delta$. The sum of all these rectangles is the estimate. For a well behaved function $f$, we expect the limit of the sums of these rectangles to equal the $\int_a^b f(x)dx$ as $n$ gets large.

Write a function

```
double integrate(double fun(double), double a, double b, int n)
```

That will use this strategy to compute the definite integral of a function when the interval is divided into $n$ subintervals. The function will return the sum of the areas of the rectangles under the function.

Comment out, but do not remove, the code for printing the values of a function at the beginning of each subinterval. As before, ask the user to enter the bounds $a$ and $b$ for the interval. But this time, loop 5 times, each time asking the user for a different integer to use for the number of subintervals.

Here is a sample output

```
Enter the lower and upper bounds of an interval: 0 5
Enter the number of subintervals to use: 500
The estimated integral for the identity function is 12.525
The estimated integral for the squaring function is 41.7917
Enter the number of subintervals to use: 1000
The estimated integral for the identity function is 12.5125
The estimated integral for the squaring function is 41.7292
Enter the number of subintervals to use: 2000
The estimated integral for the identity function is 12.4937
The estimated integral for the squaring function is 41.6354
Enter the number of subintervals to use: 4000
The estimated integral for the identity function is 12.4969
The estimated integral for the squaring function is 41.651
Enter the number of subintervals to use: 8000
The estimated integral for the identity function is 12.4984
The estimated integral for the squaring function is 41.6589
```

## 4. Reflection

Work out by hand the definite integrals $\int_a^b x dx$ and $\int_a^b x^2 dx$. Observe that as the number of subintervals increases, we get better and better approximations to the values of these integrals.

## 5. The area of a unit circle

The circle of radius 1 centered at the origin has equation $x^2 + y^2 = 1$. The top half of the circle has equation

$$y = \sqrt{1 - x^2}$$

Thus the integral

$$\int_{-1}^{1} \sqrt{1 - x^2} dx$$

gives the area of the top half of the unit circle, and the integral

$$\int_{-1}^{1} 2\sqrt{1 - x^2} dx$$

gives the area of the unit circle.

Add code that computes the above integral. Comment out the rest of the code in main (but do not delete it) and add code that allows the user to compute the this integral over $[-1, 1]$ for 5 different numbers of subintervals:

```
Enter the lower and upper bounds of an interval: -1 1
Enter the number of subintervals to use: 500
The estimated integral for the top circle function is 3.1413
Enter the number of subintervals to use: 1000
The estimated integral for the top circle function is 3.14149
Enter the number of subintervals to use: 2000
The estimated integral for the top circle function is 3.14156
Enter the number of subintervals to use: 4000
The estimated integral for the top circle function is 3.14158
Enter the number of subintervals to use: 8000
The estimated integral for the top circle function is 3.14159
```

## 6. More Reflection

Can you explain why the value of this integral is what it is?

## 7. Due date

This lab is due Saturday night at the end of Week 10.