CSCE 210 DESIGN AND ANALYSIS OF ALGORITHMS
STUDY GUIDE 2

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Here are some exercises to help you prepare for the next quiz. Work these problems and ask for help if you need it.

1. Part A

Do the following exercises from the course textbook.

Exercise 2.5 (a), (b), (c), (d), (e), (j).

Exercises 2.16, 2.17, 2.23(a), 2.23(b).

More parts will be added after Tuesday and Thursday lectures.

2. Part B

1. We noted in class that matrices are a generalization of numbers. Because there is a divide and conquer algorithm for number multiplication, it makes sense to look for a divide and conquer algorithm for matrix multiplication.

You can also think of polynomials as a generalizations of numbers. Like numbers and matrices, we can perform both addition and multiplication of polynomials.

A polynomial

$$P(X) = a_n X^n + a_{n-1} X^{n-1} + \cdots + a_1 X + a_0$$

is called an integer polynomial of degree n if all the coefficients a_i are integers. Such a polynomial is also written $P(X) = \sum_{k=0}^{n} a_k X^k$.

(a) Show that two polynomials $P(X) = \sum_{k=0}^{n} a_k X^k$ and $Q(X) = \sum_{k=0}^{n} b_k X^k$ can be added in linear time, and argue that linear time is the best possible.

(b) Show that the usual method of multiplying two polynomials of degree n requires $O(n^2)$ time.

(c) Given a polynomial $P(X) = \sum_{k=0}^{n} a_k X^k$ of degree n, we can express the polynomial in terms of two polynomials of degree $\frac{n}{2}$ (using integer division):

$$P(X) = [P_L(X)]\, X^{n/2} + P_R(X).$$

Let

$$P(X) = 23X^4 + 15X^2 - 5X + 2$$

and let

$$Q(X) = -5X^4 + X^3 - 13.$$

Express both $P(X)$ and $Q(X)$ as a sum that involves polynomials of degree 2 $P_L(X), P_R(X), Q_L(X)$ and $Q_R(X)$ and identify those four polynomials.

(d) Express the polynomial product $P(X)Q(X)$ in terms of sums of products of the four smaller polynomials $P_L(X), P_R(X), Q_L(X)$ and $Q_R(X)$.

(e) On the basis of all this, describe a divide and conquer algorithm for multiplying polynomials of degree n and write a recurrence relation that describes the complexity of that algorithm.

(f) Solve the recurrence relation to show that the divide and conquer algorithm requires $O(n^2)$ time.

2. Let A be an $m \times n$ matrix, and let B be an $n \times p$ matrix. Let a_{ij} be the entry in row i and column j of A, and similarly define b_{ij} to be the entry in row i and column j of B.

(a) How many rows and columns does the product D of A and B have?

(c) Write a method

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int getProductEntry(A[1..m][1..n], B[1..n][1..p], r, c)
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that returns the entry in row r and column c of the matrix product of A and B.

(d) Compute the matrix product

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ a & b & c \end{pmatrix} \begin{pmatrix} x & 0 & 0 \\ y & 1 & 1 \\ z & 0 & 1 \end{pmatrix}$$