

## CSCE 210 QUIZ 1 (RECURSION)

### SOLUTIONS

1. Complete the following recursive method for summing a range of an array  $a[L \dots U]$  and  $0 \leq L \leq U$  where  $U$  is less than the size of the array. In this case the recursive method should return the sum of all array entries  $a[L], a[L + 1], \dots, a[U]$ .

You may not modify the part of the method already given.

```
int sum(int [] a, int L, int U)
{
    if (L > U) return 0;
    if (L == U) return a[L];
    // put the rest of the code here

}
```

SOLUTION:

For the non-base case, recursively compute the sum of the array entries

$$a[L+1], \dots, a[U-1]$$

and then add the two entries at the end of the sub-range, namely  $a[L]$  and  $a[U]$ .

```
int sum(int [] a, int L, int U)
{
    if (L > U) return 0;
    if (L == U) return a[L];

    return a[L] + a[U] + sum(a, L+1, U-1) ;
}
```

2. Complete the following recursive method for computing the minimum value of a segment of an array  $a[0 \dots U]$ <sup>1</sup>. For example, if the array is

```
array values: [ 34, 23, 12, 78, 100 ]
array indices: 0  1  2  3  4
```

and the call is `minimum(a, 1)` then the method returns 12. However, if the call is `minimum(a, 3)` then the call returns 78.

```
int minimum( int [] a, int U)
{
    // put the rest of the code here

}
```

SOLUTION:

Unfortunately this question has a typographical error. The question should have asked for computing the minimum value of a segment  $a[L \dots a.length - 1]$  of the array, instead of  $a[0 \dots U]$ .

As a result of the typo, the question was deleted from the quiz and everybody got 5 free points.

Try and solve the corrected problem. The method would be

```
int minimum( int [] a, int L)
{
    // put the rest of the code here

}
```

---

<sup>1</sup>This should have been  $a[L \dots a.length - 1]$ .

3. Complete the following recursive method for printing the segment of a string from a given index to the end of the string. For example, the call

```
printstring("NorthCentral", 3);
```

will print "thCentral" while the call `printstring("North", 5)`; will print the empty string.

HINT: you can use `System.out.print(str.charAt[k])` to print the single character at position `k` of the string `str`.

```
void printstring(String str, int L)
{
    // place your code here
}
```

SOLUTION:

The base case is when the parameter `L` is equal to the length of the string: in this case there is nothing to do. So we only have something to do for the non-base case, which is when `L < str.length()`.

```
// print the part of the string from
// L until the end of the string.

void printstring(String str, int L)
{
    if (L < str.length())
    {
        // print the character at L
        System.out.print(str.charAt[L]);

        // recursively print the part of the string
        // from L+1 until the end.
        printString(str, L+1);
    }
}
```

4. In the Quicksort algorithm, we make use of a method called

```
int partition(int [] arr, int lower, int upper)
```

which returns the index of an element called the *pivot*. Suppose the method returns an integer  $p$  where the value of `arr[p]` is  $X$ .

(a) What can you say about the values `arr[k]` when  $\text{lower} \leq k < p$ ?

(b) What can you say about the values `arr[k]` when  $p+1 \leq k < \text{upper}$ ?

SOLUTION:

(a) `arr[k] < X`.

Notice that saying that these values are to the left of  $p$ , or saying that they are between `lower` and  $p$ , is just repeating or rephrasing the information

$$\text{lower} \leq k < p$$

which is already given in the question.

The correct answer avoids confusing  $p$  and `arr[p]`. The former is an index position into the array, the latter is the value of the array at index position  $p$  and is equal to  $X$ .

Similar considerations apply in part (b).

(b) `arr[k] >= X`.

5. Assuming that you have a method `partition` that works as described in Question 4, complete the following code for the recursive Quicksort method:

```
void Quicksort(int [] arr, int L, int U)
{

}
}
```

SOLUTION:

```
void Quicksort(int [] arr, int L, int U)
{
    if (L < U)
    {
        int p = partition(arr, L, U);
        Quicksort(arr, L, p-1);
        Quicksort(arr, p+1, U);
    }
}
```

Some of the mistakes made were:

(1) Forgetting the base case.

(2) Putting the call to

```
int p = partition(arr, L, U);
```

outside of the base case.

(3) Including type information in a method call, as in

```
Quicksort(int [] arr, int L, int p-1);
```

Type information belongs in a method definition, never in a method call.

(4) Not knowing whether to use `p`, `p+1`, or `p-1` in the recursive call to `Quicksort`.

(5) Not studying the lecture notes on Quicksort before the quiz.