CSC 231 HOMEWORK 6

PROFESSOR GODFREY C. MUGANDA

You must show your work to get credit.

1. Suppose that $\dfrac{1}{(1-4x)(1-3x)} = \dfrac{a}{1-4x} + \dfrac{b}{1-3x}$ for all real number values of x. Determine the constants a and b.

2. Express $\dfrac{2x-3}{(x-2)(x-3)}$ as a sum of partial fractions.

3. Expand the generating function $\dfrac{x}{(x-1)(x-2)}$ into the form $\sum_{k=0}^{\infty} a_k x^k$. Hint: Use Table 1 on page 542 of the class textbook.

4. Expand the generating function $\dfrac{x}{(x-1)^2}$ into the form $\sum_{k=0}^{\infty} a_k x^k$. Hint: Use Table 1 on page 542 of the class textbook.

5. Consider the recurrence relation

$$a_n = 7a_{n-1} - 12a_{n-2} \quad a_0 = 3 \text{ and } a_1 = 11$$

(a) Determine the terms a_2 and a_3.
(b) Determine the characteristic equation of this recurrence relation and solve it to determine its two roots.
(c) Use the two roots of the characteristic equation to determine a closed-form formula for a_n.

6. Consider the recurrence relation

$$a_n = 7a_{n-1} - 12a_{n-2} \quad a_0 = 3 \text{ and } a_1 = 11$$

(a) Show how to obtain the generating function for the sequence of terms a_n as a rational function (fraction of polynomials)

$$G(x) = \frac{r(x)}{d(x)}$$

where $r(x)$ and $d(x)$ are polynomials of degree 1 and 2, respectively.
(b) Factor $d(x)$.
(c) Express $G(x)$ as a sum of partial fractions whose denominators are polynomials of degree 1.
(d) Using the expression you derived in part (c) and the tables of common generating functions in Table 1 on page 542 of the class textbook to express $G(x)$ as an infinite series.
(f) Use the answer to part (d) to determine the terms a_n of the sequence defined by the recurrence relation.

Due date: This is due Thursday of Week 8. However, it is a good idea to get these done before Wednesday, as similar problems will be on the test.