CSC 231 HOMEWORK 3
FALL 2015

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1. More examples of Induction Proofs

Here is a proof of the Exercise 35 on page 330. The proof we did in class did not really use induction. This one does.

35. Prove that $n^2 - 1$ is divisible by 8 whenever n is an odd positive integer.

As in class, we make the transformation $n = 2k + 1$, where k is a non-negative integer, so that the problem becomes to prove the statement $P(k)$ that

$$(2k + 1)^2 - 1 \text{ is divisible by 8 whenever } k \geq 0.$$

It is easy to show that $P(0)$ is true. Let us assume that $P(k)$ is true and show that $P(k + 1)$ must also be true.

Note that the statement $P(k + 1)$ is

$$(2k + 3)^2 - 1 \text{ is divisible by 8 whenever } k \geq 0.$$

To prove this by induction we need to somehow see the expression $(2k + 1)^2 - 1$ in $(2k + 3)^2 - 1$. To this end, we write $2k + 3$ as $2k + 1 + 2$. We get

$$\begin{aligned}(2k + 3)^2 - 1 &= ((2k + 1) + 2)^2 - 1 \\ &= (2k + 1)^2 + 4(2k + 1) + 4 - 1 \\ &= (2k + 1)^2 - 1 + 4(2k + 1) + 4 \\ &= (2k + 1)^2 - 1 + 8k + 8\end{aligned}$$

which shows that $(2k+3)^2-1$ is divisible by 8 when we use the induction hypothesis.

2. Homework 3 Assignment

Homework 3 is due Friday at the end of Week 4.

1. Do problems 10, 14 and 20 on page 330.

2. Write a recursive method to solve the problem of expressing any fraction expressed as a ratio of two positive integers as a sum of the *fewest* fractions all of which have numerator equal to 1. Your program will take as input two positive integers p and q and output a list of the denominators of all fractions with numerator 1 that add up to $\frac{p}{q}$.

For example,

$$\frac{3}{2} = \underset{1}{\frac{1}{1}} + \frac{1}{2}$$

So your program will print the list 1, 2.

Your program should keep asking for input for p and q until a value of 0 is entered for at least one of p and q. At that point the program terminates.

The recursive method that solves the problem must have the following signature

```
static void  breakUp(int p, int q, List<Integer> denoms)
```

The method is passed a list of integers, and it adds all denominators of the numerator 1 fractions whose sum makes up the fraction $\frac{p}{q}$.

Here is a sample run of the type of output your program should produce:

```
Enter numerator p: 3
Enter denominator q: 7
[3, 11, 231]
Enter numerator p: 1
Enter denominator q: 2
[2]
Enter numerator p: 4
Enter denominator q: 5
[2, 4, 20]
Enter numerator p: 0
Enter denominator q: 0
```

3. Group Project

A group project will be assigned in class on Tuesday. Everybody should make every effort to be in class on Tuesday so you can arrange to be in group. Groups will be limited to a maximum size of 3 and a minimum size of 2.

Group project will be due Monday of Week 5.